1. Suppose that x satisfies the inequality $|x^2 - 2| > 1$. Which statement must be true?

   (a) x must satisfy $x > \sqrt{3}$.

   (b) x must satisfy $x > \sqrt{3}$ or $x < -\sqrt{3}$.

   (c) x must satisfy $-1 < x < 1$.

   (d) x must satisfy $x > \sqrt{3}$, $x < -\sqrt{3}$, or $-1 < x < 1$.

   (e) None of the above statements is true.

2. Let R be the region bounded by $y(t) = \dfrac{\cos(\sqrt{t})}{\sqrt{t}}$, $t = 0$, and $y = 0$ in the 1st quadrant. Find the area of R.

   (a) 1

   (b) 2

   (c) $\dfrac{\pi}{2}$

   (d) π

   (e) 2π

3. Consider the following statement:

   If condition P and condition Q are both true, then condition R is true.

   Which of the following statements is equivalent to the above statement?

   (a) If either condition P or condition Q is true, then condition R is true.

   (b) If condition P is false, then condition R is false.

   (c) If condition P is false and condition Q is false, then condition R is false.

   (d) If condition R is false, then either condition P or condition Q is false.

   (e) If condition R is false, then both conditions P and Q are false.

4. Suppose the probability of Team A winning a single game against Team B is $\frac{2}{3}$. What is the probability of Team B winning at least 1 game out of 3 games?

   (a) $\frac{1}{27}$
   (b) $\frac{1}{9}$
   (c) $\frac{8}{27}$
   (d) $\frac{1}{3}$
   (e) $\frac{19}{27}$

5. Evaluate

$$1 + 2 + 4 + 8 + 16 + ... + 512 + 1024.$$

   (a) 2043
   (b) 2045
   (c) 2047
   (d) 2049
   (e) 2051

6. Evaluate $\sin\left(\frac{\pi}{12}\right)$.

   (a) $\frac{\sqrt{6}-\sqrt{2}}{4}$
   (b) $\frac{\sqrt{2}-\sqrt{6}}{4}$
   (c) $\frac{\sqrt{6}+\sqrt{2}}{4}$
   (d) $\frac{\sqrt{6}+\sqrt{3}}{4}$
   (e) $\frac{\sqrt{6}-\sqrt{3}}{4}$

7. Suppose that a recent poll reveals that 52% of college students watch football at least once a month, 35% of college students watch tennis at least once a month, and that 12% watch both football and tennis at least once a month. What percentage of college students watch neither football nor tennis at least once a month?

   (a) 1%
   (b) 13%
   (c) 24%
   (d) 53%
   (e) 88%

8. Locate all the vertical asymptotes for the function

$$f(x) = \frac{x^3 - 7x^2 + 16x - 12}{x^2 - 5x + 6}.$$

   (a) $x = 2$
   (b) $x = 3$
   (c) $x = 2$ and $x = 3$
   (d) $x = 2$, $x = 3$, and $x = 6$
   (e) $f(x)$ has no vertical asymptotes.

9. Suppose a ferris wheel moves at a speed of 2 revolutions per minute. If the distance between any 2 carriages on opposite sides of the wheel is 50 feet, find the linear speed of any of these carriages.

   (a) $\frac{25}{2\pi}\frac{\text{feet}}{\text{minute}}$
   (b) $\frac{25}{4\pi}\frac{\text{feet}}{\text{minute}}$
   (c) $25\pi\frac{\text{feet}}{\text{minute}}$
   (d) $50\pi\frac{\text{feet}}{\text{minute}}$
   (e) $100\pi\frac{\text{feet}}{\text{minute}}$

10. Evaluate

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{\sqrt{k}}{n^{3/2}}.$$

(a) 0

(b) $\frac{1}{3}$

(c) $\frac{2}{3}$

(d) 1

(e) The limit does not exist.

11. Evaluate $\cos\left(2\sin^{-1}\left(\frac{1}{5}\right)\right)$.

(a) $\frac{\sqrt{24}}{5}$

(b) $\frac{24}{25}$

(c) $\frac{\sqrt{23}}{5}$

(d) $\frac{23}{25}$

(e) $\frac{1}{5}$

12. Let $g(x)$ be a positive continuous function defined for all x and let $f(x) = \int_{-1}^{x^2} g(t)dt$. Find the interval where $f(x)$ is increasing.

(a) $(-\infty, 0)$

(b) $(0, \infty)$

(c) $(-\infty, -1)$

(d) $(-1, \infty)$

(e) $(-\infty, \infty)$

13. Calculate the median for the following data set:

$$7, 2, 4, 2, 6, 15$$

(a) 2
(b) 3
(c) 4
(d) 5
(e) 6

14. Suppose both the following statements are true:

I. If either Jack or Jill play on Team A, then Team A will win the game.
II. If it is either cold or rainy, then Team A will lose the game.

Suppose Team A wins the game. What can be concluded?

(a) Jack and Jill both played on Team A.
(b) Either Jack or Jill played on Team A.
(c) It was cold and rainy.
(d) It was either not cold or not rainy.
(e) None of the above can be concluded.

15. Suppose ACT scores follow a normal distribution with a mean of 21 and standard deviation of 4. From the list below, what is the minimum score that would fall in the 97th percentile?

(a) 29
(b) 30
(c) 31
(d) 32
(e) 33

16. How many solutions are there for

$$\cos^2(2x) = \sin^2(2x) + 1$$

in the interval $[0, 2\pi]$?

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5

17. Evaluate

$$\lim_{x\to\infty} \left(\sqrt{x^2+4x+1} - \sqrt{x^2+16x}\right)$$

if the limit exists.

(a) 0
(b) -2
(c) -6
(d) -12
(e) The limit does not exist.

18. Let $A(x)$ represent the area of an equilateral triangle with sidelength x. Find a formula for $A(x)$.

(a) $A(x) = \dfrac{x^2}{2}$
(b) $A(x) = \dfrac{x^2}{4}$
(c) $A(x) = \dfrac{\sqrt{3}x^2}{2}$
(d) $A(x) = \dfrac{\sqrt{3}x^2}{4}$
(e) $A(x) = \dfrac{\sqrt{3}x^2}{8}$

19. Define the sequence $\{a_n\}$ recursively by

$$a_0 = 2^{2020}$$

$$a_n = \frac{a_{n-1}}{2} + 1, \quad n \geq 1.$$

Evaluate a_{2019}.

(a) $\dfrac{2^{2018} - 2}{2^{2016}}$

(b) $\dfrac{2^{2019} - 2}{2^{2017}}$

(c) $\dfrac{2^{2020} - 2}{2^{2018}}$

(d) $\dfrac{2^{2021} - 2}{2^{2019}}$

(e) $\dfrac{2^{2022} - 2}{2^{2020}}$

20. Cans in the shape of a right circular cylinder are being manufactured such that the metal used costs \$0.002 per squared centimeter. Find the cost (to the nearest cent) of manufacturing such a can (including the top and bottom of the can) if the radius is 3 centimeters and the height is 12 centimeters.

(a) \$0.36
(b) \$0.41
(c) \$0.48
(d) \$0.52
(e) \$0.57

21. Suppose a property is defined as follows:

There exists x such that for all y, there exists $k > y$ with $|a_k| \geq x$.

What does it mean for this property NOT to hold?

(a) For every x, there exists y such that if $k > y$ then $|a_k| \geq x$.
(b) For every x, there exists y such that if $k \leq y$ then $|a_k| < x$.
(c) For every x, there exists y such that if $k \leq y$ then $|a_k| \geq x$.
(d) For every x, there exists y such that if $k > y$ then $|a_k| < x$.
(e) For every x, there exists y such that for all $k \leq y$, we have $|a_k| < x$.

22. Define $g(t) = \begin{cases} t^a e^{-1/t}, & \text{if } t > 0 \\ 0, & \text{if } t \leq 0. \end{cases}$
Find the largest range of a for which g is differentiable.

(a) $0 \leq a < \infty$
(b) $1 \leq a < \infty$
(c) $2 \leq a < \infty$
(d) $-\infty < a < \infty$
(e) g is not differentiable for any a.

23. Suppose a game at a carnival consists of turning over a card to win the cash amount shown on that card. There are 10 total cards for which 8 show \$0, 1 shows \$2, and 1 shows \$5. What are the expected winnings on each play?

(a) \$0.00
(b) \$0.20
(c) \$0.70
(d) \$2.00
(e) \$5.00

24. How many solutions (in radians) exist for $\tan(x) = x$ in $-4\pi \leq x \leq 4\pi$?

(a) 4
(b) 5
(c) 6
(d) 7
(e) 8

25. Simplify the expression $\dfrac{1}{\sin^2(x)\sec(2x)}$.

(a) $\csc(2x)\cos(2x)$
(b) $\csc^2(x)\sec^2(x)$
(c) $\csc(x)$
(d) $\csc^2(x) - 1$
(e) none of the above

26. Define the sequence a_n by the following pattern:

$$2, \frac{3}{2}, \frac{7}{6}, \frac{25}{24}, \frac{121}{120}, \ldots$$

Simplify

$$\frac{a_{n+1}}{a_n}.$$

(a) $n+1$

(b) $\dfrac{n+1}{n}$

(c) $\dfrac{1+n!}{1+(n+1)!}$

(d) $\dfrac{1+(n+1)!}{1+n+(n+1)!}$

(e) $\dfrac{2(n+1)!}{1+(n+1)!}$

27. Let $f(x) = (1-x)^{\sqrt{3}}$. Find the angle of inclination θ (in radians) that the tangent line to $f(x)$ at $x = 0$ makes with the positive x-axis.

(a) $\theta = \dfrac{\pi}{6}$

(b) $\theta = \dfrac{\pi}{3}$

(c) $\theta = \dfrac{2\pi}{3}$

(d) $\theta = \dfrac{3\pi}{4}$

(e) $\theta = \dfrac{5\pi}{6}$

28. Let $S(n,a)$ be the sum of the coefficients for the polynomial $p(x) = (x+a)^n$. Evaluate

$$S(6,3) - S(6,2).$$

(a) 897

(b) 1,901

(c) 2,435

(d) 2,877

(e) 3,367

29. Suppose we have an isosceles triangle T with base length b and leg length a. Let T' be the triangle formed by connecting each midpoint of a side to the midpoints corresponding to the other 2 sides. Find the area of T'.

   (a) $\dfrac{ab}{4}$

   (b) $\dfrac{ab}{8}$

   (c) $\dfrac{a^2b}{8}$

   (d) $\dfrac{3ab}{16}$

   (e) $\dfrac{3a^2b}{16}$

30. Suppose a rock is dropped off a 1,000 meter building such that the acceleration $a(t)$ of the object follows $a(t) = -10 \ \frac{\text{meters}}{\text{second}^2}$. To the nearest second, how long will it take for the rock to hit the ground?

   (a) 14

   (b) 15

   (c) 16

   (d) 17

   (e) 18