1. Suppose that $P(n)$ represents some condition on the set of natural numbers $\{1, 2, 3, 4, ...\}$. Which set of conditions guarantees that $P(n)$ is true for any natural number?

   (a) $P(n)$ implies $P(n+1)$ is true.
   (b) $P(1)$ is true, $P(2)$ is true, and $P(n)$ implies $P(n+2)$ is true.
   (c) $P(1)$ is true, $P(3)$ is true, and $P(n)$ implies $P(n+2)$ is true.
   (d) $P(1)$ is true, $P(2)$ is true, and $P(n)$ implies $P(n+3)$ is true.
   (e) $P(1)$ is true and $P(n)$ implies $P(n-1)$ is true.

2. Let R be the region bounded by $y(t) = \dfrac{\ln(t)}{t}$, $t = e$, and $y = 0$. Find the area of R.

   (a) 1
   (b) $\ln(2)$
   (c) $\dfrac{1}{2}$
   (d) $\dfrac{e^2}{2}$
   (e) $e - 1$

3. Consider the following statement:

   If condition P is true, then at least one of conditions Q or R is true.

   Which of the following statements is equivalent to the above statement?

   (a) If condition P is true and condition Q is true, then condition R is true.
   (b) If condition P is true and condition Q is false, then condition R is true.
   (c) If condition P is true and condition Q is false, then condition R is false.
   (d) If condition P is false and condition Q is false, then condition R is true.
   (e) If condition P is false and condition Q is false, then condition R is false.

4. Suppose there is a bag containing 3 black balls, 4 green balls, and 5 brown balls. If two balls are drawn randomly (without replacement of the 1st ball), what is the probability of drawing at least 1 black ball?

   (a) $\frac{1}{22}$
   (b) $\frac{9}{44}$
   (c) $\frac{1}{4}$
   (d) $\frac{15}{33}$
   (e) $\frac{1}{2}$

5. Suppose that
$$y(x) = 1 + x + x^2 + ... + x^n.$$
   Evaluate and simplify $y(2)$.

   (a) $y(2) = 2^{n+1} + 2$
   (b) $y(2) = 2^{n+1} + 2^{n-1}$
   (c) $y(2) = 2^{n+1} - 1$
   (d) $y(2) = 2^{n+1} - 2$
   (e) $y(2) = 2^{n+2}$

6. Evaluate $\sec^2\left(\frac{\pi}{8}\right)$.

   (a) $\frac{2\sqrt{2}}{\sqrt{2}+1}$
   (b) $\frac{2\sqrt{2}}{1-\sqrt{2}}$
   (c) $\frac{2}{\sqrt{2}+1}$
   (d) $\frac{2}{1-\sqrt{2}}$
   (e) $\frac{\sqrt{2}}{\sqrt{2}+1}$

7. Suppose that a given exam for a disease tests accurately 90% of the time. Also suppose the population of town A is 10,000 and it carries 100 infected people. To the nearest percent, what is the approximate probability that a person who tests positive is actually infected?

   (a) 1%
   (b) 8%
   (c) 9%
   (d) 10%
   (e) 90%

8. The intensity $R(i)$ of an earthquake measured on the Richter scale is given by $R(i) = \log\left(\frac{i}{i_0}\right)$, where i is the intensity of the ground motion of the earthquake and i_0 is the intensity of the ground motion of the zero earthquake. Find the measure on the Richter scale for an earthquake with 10 times as much ground motion than an earthquake measuring 6 on the Richter scale.

   (a) 7
   (b) 8
   (c) 9
   (d) 9.5
   (e) 10

9. Let θ_0 be the radian measure of the angle corresponding to a circle with radius 4 and arc length 6π. Evaluate $\sin^{-1}(\sin(\theta_0))$.

   (a) $\frac{2}{3\pi}$
   (b) $\frac{3\pi}{2}$
   (c) $\frac{\pi}{2}$
   (d) $-\frac{\pi}{2}$
   (e) $-\frac{2}{3\pi}$

10. Calculate the following sum:

$$1 + 3 + 5 + 7 + ... + 993 + 995 + 997 + 999.$$

(a) $200,500$
(b) $225,000$
(c) $250,000$
(d) $275,000$
(e) $300,500$

11. Evaluate $\cot\left(\csc^{-1}\left(\frac{9}{5}\right)\right)$.

(a) $\frac{5}{9}$
(b) $\frac{9}{5}$
(c) $\frac{12}{\sqrt{56}}$
(d) $\frac{\sqrt{56}}{12}$
(e) $\frac{\sqrt{56}}{5}$

12. Find the equation of the line that is tangent to $f(x) = 2^x$ at $x = 1$.

(a) $y = \ln(4)(x - 1) + 2$
(b) $y = 2(x - 1) + 2$
(c) $y = \ln(2)(x - 1) + 2$
(d) $y = 2(x - 1) + \ln(2)$
(e) $y = \ln(4)(x - 1) + \ln(2)$

13. Calculate the sample standard deviation for the following data set:

$$2, 4, 5, 5$$

(a) 1

(b) $\sqrt{\frac{3}{2}}$

(c) $\sqrt{2}$

(d) 2

(e) 3

14. Suppose both the following statements are true:

I. If it is sunny and warm, then John is happy.

II. If it is Monday, then John is not happy.

Suppose John is not happy. What can be concluded?

(a) It is Monday.

(b) It is Monday, but either not sunny or not warm.

(c) It is Monday, but neither sunny nor warm.

(d) It is either not sunny or not warm.

(e) It is neither sunny nor warm.

15. Suppose ACT scores follow a normal distribution with a standard deviation of 3.3. What must be the value of the mean so that only approximately 2.5% of test takers score below 18?

(a) 21.3

(b) 22.6

(c) 23.9

(d) 24.6

(e) 27.9

16. Simplify the expression $2\sin(x)2\cos^3(x) - 2\sin^3(x)2\cos(x)$.

(a) $\cos(2x)$

(b) $\sin(2x)$

(c) $\sin(2x)\cos(2x)$

(d) $\sin(4x)$

(e) none of the above

17. Evaluate $\lim_{x\to 0^+} \left(\frac{1}{x}\right)^x$, if the limit exists.

(a) 0

(b) 1

(c) e

(d) $\ln(2)$

(e) The limit does not exist.

18. Suppose a particle traverses a circular path with a constant angular velocity of 1 revolution per minute. Find the linear velocity of the particle if the radius of the path is $\frac{1}{\pi}$ centimeters.

(a) $\frac{1}{\pi}\frac{\text{cm}}{\text{min}}$

(b) $\frac{2}{\pi}\frac{\text{cm}}{\text{min}}$

(c) $\frac{1}{2}\frac{\text{cm}}{\text{min}}$

(d) $\pi\frac{\text{cm}}{\text{min}}$

(e) $2\frac{\text{cm}}{\text{min}}$

19. Three men are told to stand in a straight line, one in front of the other. A hat is put on each of their heads. They are told that each of these hats was selected from a group of five hats: two black hats and three white hats. The first man, standing at the front of the line, can't see either of the men behind him or their hats. The second man, in the middle, can see only the first man and his hat. The last man, at the rear, can see both other men and their hats. The last man and middle man are asked in succession if they can deduce the color of his own hat to which both cannot. What must be true?

(a) The first man knows that his hat is black.

(b) The first man knows that his hat is white.

(c) The first man would know the color of his hat only if he knew the color of the last man's hat.

(d) The first man would know the color of his hat only if he knew the color of the middle man's hat.

(e) The first man does not have sufficient information to know the color of his hat.

20. An inverted right conical tank with radius 2 meters and height 5 meters is filled with water until the height of the water level reaches 3 meters. What is the radius of the cone formed from the water?

(a) 2

(b) $\frac{2}{5}$

(c) $\frac{5}{6}$

(d) $\frac{6}{5}$

(e) $\frac{3}{2}$

21. Suppose a property is defined as follows:

For every x, there exists y such that if $z > y$ then $w_z < x$.

What does it mean for this property NOT to hold?

(a) There exists x, such that for all y we have that if $z > y$ then $w_z < x$.
(b) There exists x, such that for all y we have that if $z > y$ then $w_z \geq x$.
(c) There exists x, such that for all y we have that if $z \leq y$ then $w_z < x$.
(d) There exists x, such that for all y we have that if $z \leq y$ then $w_z \geq x$.
(e) none of the above

22. Define $g(t) = \begin{cases} 1, & \text{if } t > 0 \\ -1, & \text{if } t \leq 0. \end{cases}$
What is an antiderivative of g on $(-\infty, \infty)$?

(a) $f(t) = 0$
(b) $f(t) = |t|$
(c) $f(t) = \begin{cases} t, & \text{if } t > 0 \\ -t, & \text{if } t < 0. \end{cases}$
(d) $f(t) = \int_0^t g(x)dx.$
(e) g does not have an antiderivative on $(-\infty, \infty)$.

23. Calculate the interquartile range for the following data set:

$$2, 33, 4, 56, 16, 20, 22$$

(a) 6
(b) 13
(c) 20
(d) 29
(e) 54

24. How many solutions (in radians) exist for $\tan^8(x) = 4$ in $-\frac{3\pi}{4} \le x \le \frac{3\pi}{4}$?

(a) 0
(b) 2
(c) 4
(d) 8
(e) 16

25. Simplify the expression $2\sin(x) - 2\sin^3(x)$.

(a) $\sin(x)\cos(2x)$
(b) $\sin(2x)\cos(x)$
(c) $\sin^2(2x)$
(d) $\sin(2x)\cos(2x)$
(e) none of the above

26. Find x in the following pattern:

$$2, 3, 5, 7, 11, 13, x$$

(a) 14
(b) 15
(c) 16
(d) 17
(e) 18

27. Consider the line $y(x) = -4x+20$. For each x satisfying $0 < x < 5$, a rectangle may be formed in the first quadrant by taking the 4 vertices to be $(0, 0)$, $(0, y(x))$, $(x, 0)$, and $(x, y(x))$. Find the x-value which gives you the rectangle with the maximum amount of area enclosed.

(a) $x = \frac{3}{2}$
(b) $x = \frac{7}{4}$
(c) $x = 2$
(d) $x = \frac{9}{4}$
(e) none of the above

28. Find the sum of the coefficients in the polynomial $p(x) = (x+1)^7 - (x+1)^5$.

(a) 96
(b) 108
(c) 118
(d) 128
(e) 132

29. What is the length of the base of an isosceles triangle if the legs have length 2 and each leg has opposite angle of $\frac{\pi}{6}$ radians.

(a) 2
(b) $\sqrt{3}$
(c) 4
(d) $2\sqrt{3}$
(e) $\frac{2}{\sqrt{3}}$

30. Let $y(x)$ be a differentiable function on $(-1, 1)$ satisfying $y'(x) = \frac{2x}{\sqrt{1-x^4}}$. If $y\left(\frac{1}{\sqrt{2}}\right) = \pi$, find the value of $y(0)$.

(a) 0
(b) $\frac{\pi}{6}$
(c) $\frac{\pi}{3}$
(d) $\frac{2\pi}{3}$
(e) $\frac{5\pi}{6}$